4th Malaysian Workshop on Crash Investigation & Injury Analysis

Vehicle Dynamics, Braking & Acceleration

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  • Five basic quantities
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• Coefficient of Friction and Drag Factor
  • Coefficient of Friction
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BASIC MOTION EQUATIONS
Basic Motion Equations

- **Kinematics**: the branch of engineering mechanics which deals with the motion of bodies without consideration of forces required to maintain motion.

- The subject of kinematics applied to traffic crash reconstruction issues is often limited to five basic quantities: acceleration \((a)\), time \((t)\), distance \((d)\), initial velocity \((v_i)\), and end velocity \((v_e)\).
Five Basic Quantities

• Distance
  • a linear measurement from some point
  • SI unit: meter (m)

• Time
  • SI unit: seconds (s)

• Velocity – initial and end
  • A rate of change of distance with respect to time
  • SI unit: meter per second (m/s)

• Acceleration
  • A rate of change of velocity with respect to time
  • SI unit: meter per meter per second (m/s^2)
Three Basic Equations

1) \( a = \frac{v_e - v_i}{t} \)

2) \( d = v_i t + \frac{1}{2} at^2 \)

3) \( v_e^2 = v_i^2 + 2ad \)

- Each of the three equations has four variables.
- Therefore, by algebraic manipulation, each of the variables can be solved for each equation.
Basic problem 1

• If a vehicle decelerates from a velocity of 30 m/s to stop in 6 seconds, calculate the acceleration of the vehicle.

Solution:

\[ v_e = 0 \text{ m/s}, \quad v_i = 30 \text{ m/s}, \quad t = 6 \text{ s}, \quad a = ? \]

Using equation \[ a = \frac{v_e - v_i}{t} = \frac{0 - 30}{6} \]

\[ a = -5 \text{ m/s}^2 \]

The negative sign means the vehicle is decelerating.
A car was driven at 30 m/s (108 km/hr). Suddenly the driver realized that a traffic light turned red ahead of him. If the driver apply a brake at 70 m distance before the traffic light and decelerates at 10 m/s^2, will the vehicle stopped before or after passing the traffic light?
Solution:

\( v_e = 0 \text{ m/s}, v_i = 30 \text{ m/s}, a = 10 \text{ m/s}^2, \ d = ? \)

Using equation \( v_e^2 = v_i^2 + 2ad \),

\[
d = \frac{v_e^2 - v_i^2}{2a} \]

\[
d = \frac{(0)^2 - (30)^2}{2(10)} \]

\[d = 45m\]

Answer:
The vehicle stopped at \(70-45=35\) m before the traffic light.
COEFFICIENT OF FRICTION & DRAG FACTOR
Braking, Accelerating and Turning are all limited by?
Coefficient of Friction

• The ratio of the maximum tangential reaction force applied to an object sliding across a surface expressed as a decimal portion of the normal force

\[ \mu = \frac{F}{w} \text{ (unitless)} \]
<table>
<thead>
<tr>
<th>Term</th>
<th>Definitions</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of Friction, $\mu$</td>
<td>A number representing the resistance to slide two surfaces in contact.</td>
<td>$\mu = \frac{F}{W} = \tan \alpha$</td>
</tr>
<tr>
<td></td>
<td>The ratio of the tangential force (parallel to the surface) applied to an object sliding across a surface to the normal force (perpendicular to the surface) on the object.</td>
<td></td>
</tr>
</tbody>
</table>

where:
- $\mu$ = coefficient of friction
- $F$ = horizontal force
- $W$ = weight of the object
- $\alpha$ = angle of the plane
Coefficient of Friction
Formula

Just before the object begins to slide, the frictional force, $F_f$ applied between the contact surface is $F_{f\text{ max}}$.

Free body diagram

$\sum F_{\text{total}} = 0$

$W \sin \alpha - F_{f\text{ max}} = 0$ ; where $F_f = \mu N$

$W \sin \alpha - \mu N = 0$

$W \sin \alpha - \mu(W \cos \alpha) = 0$

$\mu = \sin \alpha / \cos \alpha$

$\mu = \tan \alpha$
An object with a weight, \( W \), rests on an incline plane (angle of the plane \( \alpha_0 \)) is prevented from sliding down because of the frictional force, \( F_f \).

\[ \Sigma F_{\text{total}} = 0 \]

If the angle of the plane is increased to \( \alpha_1 \) there will be an angle at which the object begins to slide down the plane.

\[ \alpha_1 > \alpha_0 \]

This is the angle of repose and the tangent of this angle is equal to the coefficient of friction.

\[ \tan \alpha_1 = \mu \]

where \( \mu \) is the coefficient of friction.
Types of Friction in Accident Reconstruction

- **Static friction** – defined as maximum tangential force when sliding is just beginning

- **Dynamic friction** – the frictional force available when actively sliding. This type of sliding is associated with skid marks being created on the roadway
Methods of determining Coefficient of Friction (C of F) in Accident Reconstruction

- **Drag Sled** – pulling a weighted tire across the roadway surface
  - **Advantages** - Easy and quick
  - **Disadvantages** – not very accurate
Drag sled
Coefficient of Friction

Methods of determining C of F in Accident Reconstruction

Vehicle Test Skids (real world test)

• Advantages – more accurate, may be run on actual road surface at similar velocities

• Disadvantages – full scale test, difficulty replicating the vehicle, loading and tires
Coefficient of Friction

Methods of determining C of F in Accident Reconstruction

Reference Values

• Advantages – readily available, generally accepted
• Disadvantages – only typical values not specific to particular road surface or vehicle
### Coefficients of Friction of Various Roadway Surfaces

<table>
<thead>
<tr>
<th>Description of Road Surface</th>
<th>Dry Less than 30 mph</th>
<th>Dry More than 30 mph</th>
<th>Wet Less than 30 mph</th>
<th>Wet More than 30 mph</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>From</td>
<td>To</td>
<td>From</td>
<td>To</td>
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<tr>
<td><strong>Portland Cement</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New, Sharp</td>
<td>.80</td>
<td>1.20</td>
<td>.70</td>
<td>1.00</td>
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<tr>
<td>Traveled</td>
<td>.60</td>
<td>.80</td>
<td>.60</td>
<td>.75</td>
</tr>
<tr>
<td>Traffic Polished</td>
<td>.55</td>
<td>.75</td>
<td>.50</td>
<td>.65</td>
</tr>
<tr>
<td><strong>Asphalt or Tar</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>New, Sharp</td>
<td>.80</td>
<td>1.20</td>
<td>.65</td>
<td>1.00</td>
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<tr>
<td>Travelled</td>
<td>.60</td>
<td>.80</td>
<td>.55</td>
<td>.70</td>
</tr>
<tr>
<td>Traffic Polished</td>
<td>.55</td>
<td>.75</td>
<td>.45</td>
<td>.65</td>
</tr>
<tr>
<td>Excess Tar</td>
<td>.50</td>
<td>.60</td>
<td>.35</td>
<td>.60</td>
</tr>
<tr>
<td><strong>Gravel</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Packed, Oiled</td>
<td>.55</td>
<td>.85</td>
<td>.50</td>
<td>.80</td>
</tr>
<tr>
<td>Loose</td>
<td>.40</td>
<td>.70</td>
<td>.40</td>
<td>.70</td>
</tr>
<tr>
<td><strong>Cinders</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Packed</td>
<td>.50</td>
<td>.70</td>
<td>.50</td>
<td>.70</td>
</tr>
<tr>
<td><strong>Rock</strong></td>
<td></td>
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</tr>
<tr>
<td>Crushed</td>
<td>.55</td>
<td>.75</td>
<td>.55</td>
<td>.75</td>
</tr>
<tr>
<td><strong>Ice</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smooth</td>
<td>.10</td>
<td>.25</td>
<td>.07</td>
<td>.20</td>
</tr>
<tr>
<td><strong>Snow</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Packed</td>
<td>.30</td>
<td>.55</td>
<td>.35</td>
<td>.55</td>
</tr>
<tr>
<td>Loose</td>
<td>.10</td>
<td>.25</td>
<td>.10</td>
<td>.20</td>
</tr>
</tbody>
</table>
Coefficient of Friction

Effects of surface:

- Material
- Condition
- Snow
- Ice
- Water (hydroplaning)

And the effect of vehicle speed
Braking

How can we determine the initial velocity if we know the braking distance?
Braking

Easier way utilizing a specialized equation
V₁ From Braking

Kinetic Energy of the moving vehicle is converted into Work during Braking

\[ \frac{1}{2} m v^2 = F d \]

\[ m = \frac{W}{g} \quad F = W(f) \quad d = \text{distance} \]
$V_1$ From Braking

$v = \sqrt{\frac{2g}{W}} \, \text{d} \, \text{f} \, \text{W} \, \text{m/s}$

$v = \sqrt{2 \times 9.81 \, \text{d} \, \text{f}} \, \text{m/s}$

$v = \sqrt{19.62 \, \text{d} \, \text{f}} \, \text{m/s} \quad \text{(significant digits?)}$

$v = \sqrt{20 \, \text{d} \, \text{f}} \, \text{m/s}$
Braking

Assumptions:

• Closed System with Insignificant aero and other external Forces

• Constant or average values – Independent of temperature, velocity & directional orientation

• Coefficient of friction

• Drag factor

What’s a drag factor?
Drag Factor

Drag Factor Concept – a retarding force expressed as a percent of the vehicle weight.

\[ f = \frac{F}{W} \]

Allows use of the simplified brake skid formula in other situations such as when tires are not in a locked wheel skid.
Drag Factor

Also a number representing the magnitude of acceleration or braking as a decimal fraction of the acceleration of gravity.

\[ f = \frac{a}{g} \]

Also the force required to produce this amount and direction of acceleration of the subject mass.
Drag Factor

- Ratio between the acceleration and acceleration due to gravity:
  \[ f = \frac{a}{g} \]

- The force needed to produce acceleration in the same direction divided by the weight of the body to which the force is applied.

- When a vehicle slides, with all wheels locked, the coefficient of friction and the drag factor have the same value.
A force, $F$ is needed to move an object against the gravity force applied on its weight, $W$.

The ratio between these forces is defined as the drag factor.

$$f = \frac{F}{W}$$

where $F = ma$ and $W = mg$

$$f = \frac{ma}{mg} = \frac{a}{g}$$
Drag Factor

• Rolling “Friction” Losses:
  • Refers to the resisting forces that come into play when a vehicle is rolling with no braking
  • Generally, drag factor values for rolling tires are very low
    • 0.01-0.02 for non-driven wheels
    • 0.04-0.08 for driven wheels in high gear
# Drag Factor Values

## Table

<table>
<thead>
<tr>
<th>Ice</th>
<th>Snow (0.05-0.30)</th>
<th>Clean, wet paving (0.45-1.20)</th>
<th>Gravel (0.35-0.60)</th>
<th>Wet paving (0.55-0.75)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Illegal brakes</td>
<td>Fair brakes</td>
<td>Good brakes</td>
<td>Excellent brakes</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>0.40</td>
<td>0.45</td>
<td>0.50</td>
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<tr>
<td></td>
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<td>0.60</td>
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<tr>
<td></td>
<td>0.70</td>
<td>0.75</td>
<td>0.80</td>
<td>0.85</td>
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<tr>
<td></td>
<td>0.90</td>
<td>0.95</td>
<td>1.00</td>
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</tr>
<tr>
<td></td>
<td>1.20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Coefficient of friction, $\mu$ or drag factor, $f$?

- $\mu$ is used when all tyres apply brakes, causing the object to slide across the surface.
- $f$ is used when not all tyres apply brakes.

$\mu = f$ when all wheels are locked and sliding on a level surface.
Calculating Velocity from Skidmarks - Example

A vehicle skids to a stop on dry asphalt leaving 30m of skid marks. The drag factor (C of F of asphalt) is 0.7

How fast was it going before braking?
Calculating Velocity from Skidmarks - Example

How fast was the vehicle going before braking?

\[ f = 0.7 \]
\[ d = 30 \text{ m} \]

\[ v = \sqrt{20fd} \]
\[ v = \sqrt{20(0.7)(30)} \]

ANS: \( 20.5 \text{ m/s} = 73.8 \text{ km/hr} \)
Calculating Velocity from Skidmarks

Measuring Skid Marks
<table>
<thead>
<tr>
<th>Tire marks</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Skid mark/braking mark:</strong></td>
<td><strong>Yaw mark/scuff mark:</strong></td>
</tr>
<tr>
<td>Cause by locked-wheels (tire not free to rotate) due to driver applying brakes. The striations parallel to the mark.</td>
<td>Tire frictions made by a tire is rotating and sliding on road surface. Appear in oblique striations.</td>
</tr>
<tr>
<td>Gauge mark/road scars</td>
<td>Debris</td>
</tr>
<tr>
<td>------------------------------------------------------------------------------------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>Mark cause by friction of vehicle damage part (metal part) with road surface while the vehicle was moving during a collision.</td>
<td>Loose material scattered at the crash site.</td>
</tr>
</tbody>
</table>
Frequently skid marks will be of differing lengths. Which length do you use?

- Average them?
- Use the shortest?
- Use the longest?

Why?
Calculating Velocity from Skid marks

Goal is to Measure the distance that the brakes were applied.

Which is? The longest skid mark
Sample Problem: A SUV skids 30m coming to a stop. How fast was it going?

\[ v = \sqrt{20df} \]
Sample Problem: A SUV skids 30m coming to a stop. How fast was it going?

\[ D = 30 \text{m} \]

\[ f = ? \]

Need to make an assumption

Range is .6 - .8 for “traveled”

\[ f = 0.7 \text{ nominal} \]
Sample Problem: A SUV skids 30m coming to a stop. How fast was it going?

D = 30m

at \( f = 0.7 \) \hspace{1cm} v = 74 \text{ km/hr}

at min \( f = 0.6 \) \hspace{1cm} v = 69 \text{ km/hr}

at max \( f = 0.8 \) \hspace{1cm} v = 79 \text{ km/hr}

Answer: 74 km/hr (± 5 km/hr)
If it takes 30 m to stop from 25 km/hr, how many meter does it take to stop from 50km/hr?

Try to guess:
- less than 30 m? (estimate)
- 60 m?
- 80 m?
- 100 m?
- 120 m?
Answer: 120 m

Stopping distance goes up at the square of the speed
The “Power" of Energy

Many quantities in Accident Reconstruction increase at the square of speed
Vehicle Acceleration

Determined by:
1. The Driver, up to limits
2. Limited by the vehicle power
3. Limited by the C of F
4. Limited by the dynamic weight distribution on the powered wheels
• A car was driven at 30 m/s (108 km/hr). Suddenly the driver realized that a traffic light turned red ahead of him. If the driver apply a hard brake at 70 m distance before the traffic light, will the car stopped before or after passing the traffic light, if:

1) C of F is 0.7?
2) C of F is 0.4?

(Assumption: No reaction time required)
Quiz II Answer

For C of F, \( \mu = 0.7 \),

Given \( v_e = 0 \text{ m/s}, v_i = 30 \text{ m/s}, = 0.7, d = ? \)

To get \( d \),

Using equation \( v = \sqrt{20 \cdot d \cdot f} \),

\[
d = \frac{v^2}{20f} = \frac{30^2}{20 \times 0.7} = 64 \text{ m}
\]

Answer:

The vehicle stopped at (70-64) m = 6 before the traffic light.
2) For C of F, $\mu = 0.4$,

1) Given $v_e = 0$ m/s, $v_i = 30$ m/s, $\gamma = 0.7$, $d =$ ?

To get $d$,

Using equation $v = \sqrt{20 \cdot d \cdot f}$, $d = \frac{v^2}{20f} = \frac{30^2}{20 \times 0.4} = 113$ m

Answer:
The vehicle stopped at (113-70) m = 43 m after the traffic light.
Thank you